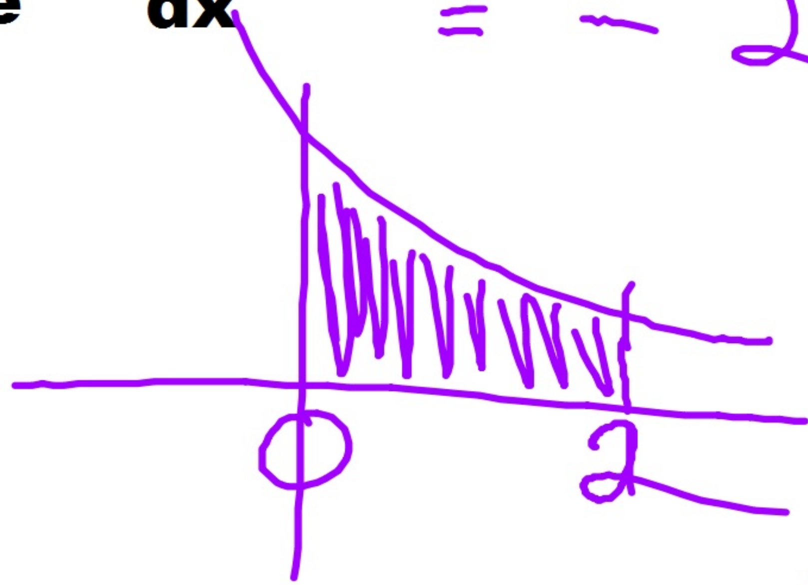
$$\int_0^2 e^{-x/2}\,dx = -2e^{-\frac{x}{2}}\Big]_0^2$$



$$\int_0^\infty e^{-\frac{x}{2}}\,dx$$

Section 8.4
Improper Integrals

Improper Integrals with Infinite Integration Limits

Integrals with infinite limits of integration are improper integrals...

1.) If f(x) is continuous on $[a,\infty)$, then

$$\int_a^{\infty} f(x)\,dx = \lim_{b\to\infty} \int_a^{b} f(x)\,dx.$$

2.) If f(x) is continuous on (-∞, b], then

$$\int_{-\infty}^{b} f(x)\,dx = \lim_{a \to -\infty} \int_{a}^{b} f(x)\,dx.$$

3.) If f(x) is continuous on (-∞,∞), then

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{c} f(x)\,dx + \int_{c}^{\infty} f(x)\,dx$$

where c is any real number.

Parts 1 and 2

If the limit is finite the improper integral CONVERGES and the limit is the value of the improper integral

If the limit fails to exist, the improper integral DIVERGES

Part 3

The integral on the left-hand side of the equation CONVERGES if both improper integrals on the right-hand side converge, otherwise it DIVERGES and has no value.

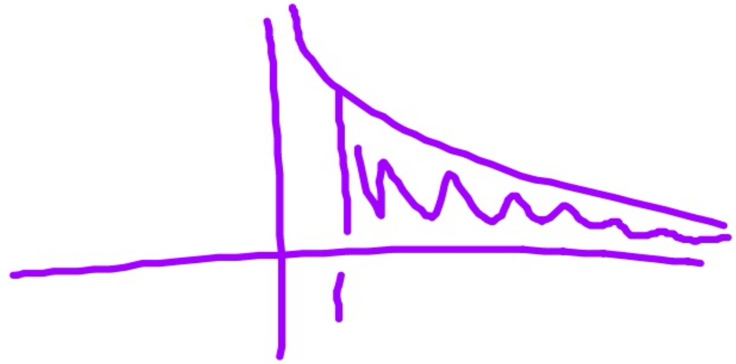
$$\int_0^{\infty} e^{-x/2}\,dx =$$

$$= \lim_{b\to\infty} \int_0^b e^{-\frac{x}{2}}\,dx$$

$$= \lim_{b\to\infty} -2e^{-\frac{x}{2}}\Big]_0^b$$

$$= \lim_{b\to\infty} \left[-2e^{-\frac{b}{2}} + 2e^{0}\right]$$

$$\int_1^\infty \frac{1}{x}\,dx =$$



$$\lim_{b\to\infty}\int_1^b \frac{1}{x}\,dx$$

diverges.

$$= \lim_{b\to\infty}\left[\ln x\right]_1^b = \lim_{b\to\infty}\left[\ln b\right]$$

$$\int_1^\infty \frac{1}{x^2}\,dx =$$

$$\lim_{b\to\infty}\int_1^b x^{-2}\,dx$$

$$\lim_{b\to\infty}\left.\frac{x^{-1}}{-1}\right]_1^b = \lim_{b\to\infty}\left.-\frac{1}{x}\right]_1^b$$

$$= \lim_{b\to\infty}\left[-\frac{1}{b}+1\right] = 1$$

Express the improper integral

$\int_{-\infty}^{\infty} e^x \, dx$ **in terms of limits of definite integrals and then evaluate the integral.**

$$\int_{-\infty}^{\infty} e^x \, dx = \int_{-\infty}^{0} e^x \, dx + \int_{0}^{\infty} e^x \, dx$$

Does the improper integral

$$\int_1^{\infty} \frac{dx}{x}$$

converge or diverge?

Evaluate

$$\int_0^{\infty} \frac{2\,dx}{x^2 + 4x + 3}$$ **or state that it diverges**

$$= \lim_{b\to\infty} \int_0^b \left(\frac{1}{x+1} + \frac{-1}{x+3} \right) dx$$

$$= \lim_{b\to\infty} \Big[\ln|x+1| - \ln|x+3| \Big]_0^b$$

Evaluate

$$\int_1^{\infty} xe^{-x}\,dx \quad \text{or state that it diverges}$$

$$= \lim_{b\to\infty} \int_1^b xe^{-x}\,dx$$

$$= \lim_{b\to\infty} \Big[-xe^{-x} + (-e^{-x})\Big]_1^b$$

Evaluate

$$\int_{-\infty}^{\infty} \frac{dx}{1+x^2} = \int_{-\infty}^{0} \frac{1}{1+x^2}\,dx + \int_{0}^{\infty} \frac{1}{1+x^2}\,dx$$

$$= \lim_{a\to-\infty} \int_{a}^{0} \frac{1}{1+x^2}\,dx + \lim_{b\to\infty} \int_{0}^{b} \frac{1}{1+x^2}\,dx$$

$$= \lim_{a\to-\infty} \tan^{-1}(x)\Big]_{a}^{0} + \lim_{b\to\infty} \tan^{-1}(x)\Big]_{0}^{b}$$

$$= \lim_{a\to-\infty} \tan^{-1}(0) - \tan^{-1}(a) + \lim_{b\to\infty} \tan^{-1}(b) - \tan^{-1}(0)$$

$$= \left(0 - -\frac{\pi}{2}\right) + \left(\frac{\pi}{2} - 0\right) = \pi$$

Rotate $y = 1/x$ from $x = 1$ to ∞ around the x-axis. Find the volume.

$$\int_1^{\infty} \frac{1}{x^p}\, dx$$

$$\int_0^{\infty} \frac{2\,dx}{x^2 + 4x + 3} =$$

Find the volume when we rotate $y = xe^{-x}$ from $[0,\infty)$ around the x-axis.

$$\int_0^5 \frac{3}{x-2}\,dx =$$

Find p for which $\displaystyle\int_0^1 \frac{dx}{x^{p+1}}$ converges

$$\int_{1}^{5} \frac{3\,dx}{x - 3} =$$

Improper Integrals with Infinite Discontinuities

Integrals of functions that become infinite at a point within the interval of integratino are improper integrals.

1.) If f(x) is continuous on (a,b], then

$$\int_a^b f(x)\,dx = \lim_{c \to a^+} \int_c^b f(x)\,dx$$

2.) If f(x) is continuous on [a,b), then

$$\int_a^b f(x)\,dx = \lim_{c \to b^-} \int_a^c f(x)\,dx$$

3.) If f(x) is continuous on [a,c] U (c,b], then

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$\int_{0}^{3} \frac{dx}{(x - 1)^{2/3}} =$$

Evaluate

$$\int_{1}^{2} \frac{dx}{x-2}$$

Test for Convergence or Divergence

Comparison Test

Let f and g be continuous on $[a,\infty)$ with $0 \le f(x) \le g(x)$ for all $x \ge a$. Then

1.) $\displaystyle\int_a^\infty f(x)\,dx$ converges if $\displaystyle\int_a^\infty g(x)\,dx$ converges

2.) $\displaystyle\int_a^\infty g(x)\,dx$ diverges if $\displaystyle\int_a^\infty f(x)\,dx$ diverges.

Does the integral $\int_1^{\infty} e^{-x^2}\,dx$ converge?

$$\int_{-\infty}^{\infty} e^x \, dx =$$

$$\int_{-\infty}^{\infty} \frac{1}{1 + x^2}\, dx =$$